Recall: Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$	$\Rightarrow \sin^2 x = 1 - \cos^2 x$ $\Rightarrow \cos^2 x = 1 - \sin^2 x$
$\sec^2 x = 1 + \tan^2 x$ *	$\Rightarrow \tan^2 x = \sec^2 x - 1$

rule: use when given an odd power of sine or cosine

Recall: Double Angle (Reduction) Formulas

$\sin 2x = 2\sin x \cos x$

given an even power of sine or cosine

$\cos 2x = 2\cos^2 x - 1$	$\Rightarrow \cos^2 x = \frac{1}{2}(\cos 2x + 1)$
$\cos 2x = 1 - 2\sin^2 x$	$\Rightarrow \sin^2 x = \frac{1}{2}(1 - \cos 2x)$

$(e^x)' = e^x$ $(\sin x)' = \cos x$ $(\ln x)' = \frac{1}{x}$

Do: $\int e^{7x}\,dx = \frac{1}{7}e^{7x} + C$

Do: $\int \cos 7x\,dx = \frac{1}{7}\sin 7x + C$

Do: $\int \frac{1}{7x-5}\,dx = \frac{1}{7}\ln|7x-5| + C$

$(\tan x)' = \sec^2 x$

then $\int \sec^2(7x)\,dx = \boxed{\frac{1}{7}\tan 7x + C}$

recall: $\int \tan x\,dx = -\ln|\cos x| + C$

then $\int \tan 2x\,dx = -\frac{1}{2}\ln|\cos 2x| + C$

recall: $\int \sec x\,dx = \ln|\sec x + \tan x| + C$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$ $(\arctan x)' = \frac{1}{1+x^2}$

Guidelines for Trigonometric Substitutions – look at format under the radical:

GIVEN:	USE:	
$\sqrt{a^2 - x^2}$	$x = a\sin\theta$ $\longrightarrow$	$dx = a\cos\theta\,d\theta$
$\sqrt{a^2 + x^2}$	$x = a\tan\theta$ $\longrightarrow$	$dx = a\sec^2\theta\,d\theta$
$\sqrt{x^2 - a^2}$	$x = a\sec\theta$ $\longrightarrow$	$dx = a\sec\theta\tan\theta\,d\theta$

GO TO P3

recall: $\int \frac{dx}{x^2\sqrt{4-x^2}}$ ← a^2

write ITO θ

$a^2 = 4 \Rightarrow a = 2$

$x = 2\sin\theta$ $\quad x^2 = 4\sin^2\theta$

$dx = 2\cos\theta\, d\theta$

$\frac{x}{2} = \sin\theta$

$\sqrt{4-x^2} \Rightarrow \sqrt{4-4\sin^2\theta}$

$= \sqrt{4}\sqrt{1-\sin^2\theta}$

$= 2\sqrt{\cos^2\theta}$

$= 2\cos\theta = \sqrt{4-x^2} \Rightarrow \cos\theta = \frac{\sqrt{4-x^2}}{2}$

$= \frac{2}{4}\int \frac{\cos\theta}{\sin^2\theta\sqrt{4-4\sin^2\theta}}\, d\theta$

$= \frac{1}{2}\cdot\frac{1}{2}\int \frac{\cancel{\cos\theta}}{\sin^2\theta\ \cancel{\cos\theta}}\, d\theta$

$= \frac{1}{4}\int \frac{1}{\sin^2\theta}\, d\theta$ $\qquad \frac{1}{\sin\theta} = \csc\theta$

$= \frac{1}{4}\int \csc^2\theta\, d\theta$

know $-(\cot\theta)' = +\csc^2\theta$

$= -\frac{1}{4}\cot\theta + C$

rewrite ITO x

$= -\frac{1}{4}\frac{\cos\theta}{\sin\theta} + C$

$= -\frac{1}{4}\frac{\frac{\sqrt{4-x^2}}{\cancel{2}}}{\frac{x}{\cancel{2}}} + C = \boxed{-\frac{\sqrt{4-x^2}}{4x} + C}$

ALTERNATE SOL'N:

$-\frac{1}{4}\cot\theta + C$

SOH CAH TOA

use $x = 2\sin\theta$

$\frac{x}{2}\frac{\text{OPP}}{\text{HYP}} = \sin\theta$

$\cot\theta = \frac{\text{ADJ}}{\text{OPP}} = \frac{\sqrt{4-x^2}}{x}$

$x^2 + \text{ADJ}^2 = 4$

$\text{ADJ}^2 = 4 - x^2$

$\text{ADJ} = \sqrt{4-x^2}$

$-\frac{1}{4}\cdot\frac{\sqrt{4-x^2}}{x} + C$

ex. $\int \frac{dx}{\sqrt{9+x^2}}$ $a^2=9 \Rightarrow a=3$

$x = 3\tan\theta$ $x^2 = 9\tan^2\theta$

$dx = 3\sec^2\theta\, d\theta$

$\sqrt{9+x^2} \Rightarrow \sqrt{9+9\tan^2\theta}$

$= \sqrt{9}\sqrt{1+\tan^2\theta}$

$= 3\sqrt{\sec^2\theta}$

from p. 1

$\sec^2\theta = 1+\tan^2\theta$

$\sqrt{9+x^2} = 3\sec\theta \Rightarrow \frac{\sqrt{9+x^2}}{3} = \sec\theta$

$\frac{x}{3} = \tan\theta$

$= 3\int \frac{\sec^2\theta}{\sqrt{9+9\tan^2\theta}}\, d\theta$

$= \frac{\cancel{3}}{\cancel{3}}\int \frac{\sec^{\cancel{2}}\theta}{\cancel{\sec\theta}}\, d\theta$

$= \int \sec\theta\, d\theta$

$= \ln|\sec\theta + \tan\theta| + C$

re-write ITO x

$= \boxed{\ln\left|\frac{\sqrt{9+x^2}}{3} + \frac{x}{3}\right| + C}$

OR

$\ln\left|\frac{1}{3}\sqrt{9+x^2} + \frac{1}{3}x\right| + C$

ex. $\int \frac{x^2}{\sqrt{25-x^2}}\,dx$

$a^2 - x^2$

$a^2 = 25 \Rightarrow a = 5$

$x = 5\sin\theta \qquad x^2 = 25\sin^2\theta$

$dx = 5\cos\theta\, d\theta$

$\sqrt{25-x^2} = 5\cos\theta \Rightarrow \cos\theta = \frac{\sqrt{25-x^2}}{5}$ $\frac{ADJ}{HYP}$

$= 25\int \frac{\sin^2\theta}{\sqrt{25-25\sin^2\theta}} \cdot 5\cos\theta\, d\theta$

Do!

$= \frac{125}{5}\int \frac{\sin^2\theta\,\cancel{\cos\theta}}{\cancel{\cos\theta}}\,d\theta$

$= 25\int \sin^2\theta\, d\theta$

$= 25\cdot\frac{1}{2}\int (1-\cos 2\theta)\, d\theta$

$= \frac{25}{2}\left(\theta - \frac{1}{2}\sin 2\theta\right) + C$

Rewrite ITO x

$= \frac{25}{2}\left(\arcsin\frac{x}{5} - \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}x\sqrt{25-x^2}}{25}\right) + C$

$$\boxed{= \frac{25}{2}\arcsin\left(\frac{x}{5}\right) - \frac{x\sqrt{25-x^2}}{2} + C}$$

use Reduction Formula

$$\boxed{\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)}$$

write θ ITO x:

$x = 5\sin\theta$

$\frac{x}{5} = \sin\theta$

$\arcsin\frac{x}{5} = \theta$

write $\sin 2\theta$ ITO x:

$\sin 2\theta = 2\sin\theta\cos\theta$ from p 1

$= 2\cdot\frac{x}{5}\cdot\frac{\sqrt{25-x^2}}{5}$

$\sin 2\theta = \frac{2x\sqrt{25-x^2}}{25}$

$- \frac{\cancel{25}}{2}\cdot\frac{x\sqrt{25-x^2}}{\cancel{25}}$

$\tan^2\theta = \sec^2\theta - 1$

ex. $\int \frac{dx}{x^2\sqrt{x^2-25}}$ $\leftarrow x^2 - a^2$

$a = 5$

$x = 5\sec\theta \qquad x^2 = 25\sec^2\theta$

$dx = 5\sec\theta\tan\theta\, d\theta$

$$= \frac{5}{25}\int \frac{\cancel{\sec\theta}\tan\theta}{\sec^{\cancel{2}}\theta\sqrt{25\sec^2\theta - 25}}\,d\theta \longrightarrow \sqrt{x^2-25} \Rightarrow \sqrt{25\sec^2\theta - 25}$$

Do: simplify to eliminate radical

$$= 5\sqrt{\sec^2\theta - 1}$$
$$= 5\sqrt{\tan^2\theta}$$
$$= 5\tan\theta$$

$$= \frac{1}{5}\cdot\frac{1}{5}\int \frac{\cancel{\tan\theta}}{\sec\theta\,\cancel{\tan\theta}}\,d\theta$$

$$= \frac{1}{25}\int \frac{1}{\sec\theta}\,d\theta \qquad \cos\theta = \frac{1}{\sec\theta}$$

$$= \frac{1}{25}\int \cos\theta\, d\theta$$

$$= \frac{1}{25}\sin\theta + C$$

Do: rewrite ITO x

$x = 5\sec\theta$

$\frac{x}{5} = \sec\theta \Rightarrow \frac{5}{x}\frac{A}{H} = \cos\theta$

opp $= \sqrt{x^2-25}$

$\therefore \sin\theta = \frac{O}{H} = \frac{\sqrt{x^2-25}}{x}$

$$= \frac{1}{25}\cdot\frac{\sqrt{x^2-25}}{x} + C$$

$$= \boxed{\frac{\sqrt{x^2+25}}{25x} + C}$$

Do: $\int \sqrt{16-x^2}\,dx$ $a=4$ $x=4\sin\theta \leftarrow$ $x^2=16\sin^2\theta$

$dx=4\cos\theta\,d\theta$

$=4\int\sqrt{16-16\sin^2\theta}\,\cos\theta\,d\theta$

$=4\int\sqrt{16(1-\sin^2\theta)}\,\cos\theta\,d\theta$

$=4\int\sqrt{16}\sqrt{1-\sin^2\theta}\,\cos\theta\,d\theta$

$=16\int\cos\theta\cos\theta\,d\theta$

$=16\int\cos^2\theta\,d\theta$

$=16\cdot\frac{1}{2}\int(\cos2\theta+1)\,d\theta$

$=8\left(\frac{1}{2}\sin2\theta+\theta\right)+C$

$=4\sin2\theta+8\theta+C$

$=\frac{\cancel{4}x\sqrt{16-x^2}}{\cancel{8}}+8\arcsin\left(\frac{x}{4}\right)+C$

$=\boxed{\frac{x}{2}\sqrt{16-x^2}+8\arcsin\left(\frac{x}{4}\right)+C}$

$\cos2\theta=2\cos^2\theta-1$

$\frac{1}{2}(\cos2\theta+1)=\frac{\cancel{2}}{\cancel{2}}\cos^2\theta$

$\frac{x}{4}=\sin\theta$

$\arcsin\left(\frac{x}{4}\right)=\theta$

$\sin2\theta=2\sin\theta\cos\theta$

$=\cancel{2}\cdot\frac{x}{\cancel{4}_{2}}\frac{\sqrt{16-x^2}}{4}$

$\sin2\theta=\frac{x\sqrt{16-x^2}}{8}$